NUMBER AND ALGEBRA

Star Sequence

Adapted from an activity in an article by van den Kieboom et al. in 'Journal of Mathematics Teacher Education', Vol 17 (5).

Learning focus

- Explore growing patterns
- Devise rules to express patterns and relationships
- Relate rules to the structure of the spatial pattern

Key vocabulary

- Pattern
- Sequence
- Growing
- Increasing
- Term
- Each time
- Always
- Rule

Resources

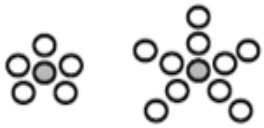
- Conkers (and other natural resources)
- Pencils, paper and clipboards
- Camera

This is the second activity in a series of three activities on algebraic reasoning. The first is 'Zeds' and the third is 'Tables and Chairs'.

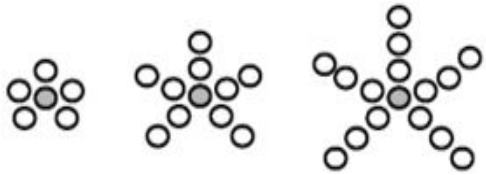
Activity

Arrange a set of conkers to form a star as shown:

This the first figure in a sequence of stars. Create the second figure by adding one more conker to each 'point'.



Repeat again for the third figure.



Ask children to describe the pattern.

How many conkers are in the first figure? Second? Third?
How is the pattern growing?
What do you notice?
What changes each time?
What stays the same?

Encourage them to predict the number of conkers needed to make the 4th and 5th figures. Once they have given their predictions, invite two children to check by using more conkers to continue the pattern.

Work with the children to complete a table to record their findings.

Term	Number of conkers
1	6
2	11
3	16
4	21
5	26

Teaching point

Keeping a systematic record helps when looking for patterns and when explaining logic and reasoning.

Ask children to describe any patterns and relationships they can see. They are most likely to use a rule which refers to the increasing number of conkers in the right hand column – a sequential generalisation.

There are 5 more conkers each time.

Add on 5 each time.

Teaching point

A sequential (or near) generalisation focuses on what changes each time in a sequence. It can be used to predict the next term in the sequence.

This is correct but encourage them to look across the columns for a rule connecting the term in the sequence and the number of conkers – a global generalisation. A rule such as 'times 5 plus 1' or 'multiply by 5 and add 1' should be suggested. Encourage the children to express this rule in a complete sentence.

The total number of conkers is always 5 times the number in the sequence (the term) plus 1.

Multiply the number in the sequence (the term) by 5 and add 1 to find the total number of conkers.

Teaching point

A global (or far) generalisation is a statement about what is the same; it does not change. It can be used to determine any term in the sequence. It is a much more powerful rule than the sequential generalisation.

The use of effective questioning can help to support the development of a global generalisation.

> *If you knew how many conkers were in the 9th figure, how could you work out how many are in the 10th figure?*
>
> *Can you work out how many conkers you would need for the 20th figure (without making the figure or drawing a picture)? Explain how you worked this out.*

Teaching point

It can be tedious to extend the pattern to a figure further out in the sequence. The use of a 'large number' challenges children to look for the relationship between the term in the sequence and the number of elements (conkers) needed to make it.

Encourage children to explain why their rule makes sense by relating it to the structure of the spatial pattern.

> *Each figure has a conker in the centre. There are 5 points and each time we add on one more conker to each point. The first figure has one conker along each point; the second figure has two conkers along each point, and so on. So the total number of conkers can be found by multiplying the position in the sequence by five and adding one (the central conker).*

Teaching point

The physical (or pictorial) construction of a pattern can be a powerful tool in enabling children to generalise relationships. It helps children to relate their rules to the structure of the spatial pattern.

Pose questions to challenge children's understanding of the growing pattern.

> *How many conkers will be needed for the 25th figure in the sequence?*
> *Will there be a figure in the sequence that is made of 100 conkers? If yes, which one? If no, why not?*
> *Will there be a figure in the sequence that is made of 206 conkers? If yes, which one? If no, why not?*

Later, children will learn how to use symbolic notation to express the rule. For example, using a to represent any term in the sequence, and b to represent the total number of conkers, the rule could be written as: $b = (5 \times a) + 1$

Taking ideas further

Invite children to use natural resources to create their own growing pattern. For example, they could create a growing pattern of crosses or a growing pattern of V-shapes:

They should be able to describe how their pattern is growing. Once they have created the first three or four terms, encourage them to record their findings in a table.

Take photographs of the different growing patterns. Invite children to share and discuss their growing patterns. Encourage them to devise rules for their growing patterns. They should also be able to explain why their rules make sense by relating them to the structure of the spatial pattern.

Later, they will learn how to express their rules using symbolic notation.

Assessment opportunities

Are the children able to:

- Continue the spatial pattern correctly
- Describe how the spatial pattern is growing using appropriate vocabulary
- Use appropriate mathematical vocabulary to express rules for growing patterns
- Explain why their rule makes sense by relating it to the structure of the spatial pattern